THERMAL RADIATION OF TURBULENT FLOWS IN THE CASE OF LARGE FLUCTUATIONS OF THE ABSORPTION COEFFICIENT AND THE PLANCK FUNCTION

V. P. Kabashnikov

UDC 535.3:532.507

An expression is obtained for the mean brightness of a flat layer with a random temperature distribution that is valid for arbitrary amplitudes of the absorption coefficient and Planck's function fluctuations. The limits of applicability are set for the approximation of optically thin fluctuations.

It is known [1] that the mean thermal radiation of turbulent flows differs from the radiation computed over mean temperature and concentration fields. The influence of turbulent fluctuations of the thermodynamic flow parameters on radiation was examined in [1-9]. However, the case of large fluctuations was not investigated in these papers.

Turbulent heated gas jets are an important class of emitters. Their main radiation, associated with deexcitation of the hottest near-axis zone, is concentrated in spectrum sections where the lengths of the quantum transits are commensurate with or exceed the transverse jet dimension. Taking into account that the external scale of turbulence l governing the correlation length of the fluctuating parameters, is an order less than the characteristic flow dimension, we obtain that the fundamental radiation occurs at frequencies for which

$$\langle k \rangle l \ll 1, \tag{1}$$

where k is the absorption coefficient, and the angular brackets denote taking the average over the ensemble of fluctuation realizations.

An optically thin fluctuation approximation (OTFA) is proposed in [1] for the analysis of the mean radiation from volumes with dimensions L satisfying the inequality

$$L \gg l, \tag{2}$$

where a spatial inhomogeneity in the temperature and concentration of fluctuation origin with dimension $\sim l$ is understood to be a pulsation. The OTFA is based on neglecting the correlation between the absorption coefficient and the radiation intensity, resulting in the following expression for the mean intensity:

$$\langle I \rangle = \int_0^L \langle kB \rangle \exp\left(-\int_0^x \langle k \rangle dy\right) dx, \tag{3}$$

where B is the Planck function.

The expression (3) contains local averages and being distinguished by sufficient simplicity describes a broad circle of situations of importance from a practical viewpoint, however, its boundaries of applicability have not yet been established.

An attempt is made in this paper to set up the OTFA limits of applicability and to determine the average radiation in a case when the mentioned approximation is not valid.

The instantaneous thermal radiation intensity is determined by the expression

$$I = \int_0^L kB \exp\left(-\int_0^x kdy\right) dx. \tag{4}$$

Denoting the fluctuating part of the random variables with a prime, we represent the mean value of the integrand in the form

$$\left\langle kB \exp\left(-\int_0^x kdy\right)\right\rangle = \langle kB \rangle \langle \tau \rangle + \exp\left(-\int_0^x \langle k \rangle dy\right) \left\langle (kB)' \exp\left(-\int_0^x k'dy\right)\right\rangle, \tag{5}$$

Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 1, pp. 35-42, July, 1985. Original article submitted June 12, 1984.

 0022-0841/85/4901-0778$09.50

where

$$\langle \tau \rangle = \exp\left(-\int_0^x \langle k \rangle \, dy\right) \left\langle \exp\left(-\int_0^x k' dy\right)\right\rangle. \tag{6}$$

The fluctuating part of the optical depth can be considered as the sum of a large number of uncorrelated components for x ≫ l, which permits considering it approximately as a normal random variable

$$\langle \tau \rangle \simeq \exp\left(-\int_0^x \langle k \rangle dy + \int_0^x \langle k'^2 \rangle \, l dy\right). \tag{7}$$

It follows from (7) that the mean transmission is determined by the mean absorption coefficient under the condition

$$\int_0^x \langle k'^2 \rangle \, l dy \ll \max\left\{1, \int_0^x \langle k \rangle \, dy\right\}. \tag{8}$$

To estimate the correlation of the emissivity (the quantity (kB)') at the point x and the transmission along the track between 0 and x, we separate the integral representing the optical depth into the sum of two integrals, one of which with the integration interval of length l adjoining the point x correlates with (kB)' while the second does not correlate with the quantity mentioned. Taking into account the smallness of the optical depth of a layer of thickness l that follows from (8) and (1), we can write

$$K \equiv \left\langle (k(x) B(x))' \exp\left(-\int_0^x k' dy\right)\right\rangle \simeq \left\langle (k(x) B(x))' \left[1 - \int_{x-l}^x k'(z) \, dz\right] \exp\left(-\int_0^{x-l} k' dy\right)\right\rangle. \tag{9}$$

Assuming its value for z = x as the characteristic value of the triple correlation in (9), we arrive at the following estimate

$$K \approx \langle (k(x) B(x))' k'(x) \rangle \, l \exp\left(\int_0^x \langle k'^2 \rangle \, l dy\right). \tag{10}$$

Comparing (10) with (5)-(7) yields the condition for which the correlation between the emissivity and the transmission

$$\langle (kB)' k' \rangle \, l \ll \langle kB \rangle \tag{11}$$

can be neglected. The following form

$$\int_0^x \langle k^2 \rangle \, l dy \ll \max\left\{1, \int_0^x \langle k \rangle \, dy\right\}, \tag{12}$$

$$\langle k^2 B \rangle \, l \ll \langle kB \rangle \tag{13}$$

can be accorded to conditions (8) and (11) by taking account of (1).

Therefore, in addition to conditions (1) and (2), the two conditions (12) and (13), which constrain the magnitude of the absorption coefficient fluctuations and of the Planck function in certain cases, must still be satisfied for the OTFA to be valid. These additional conditions are known to be satisfied for optically thin layers and start to play a greater and greater constraining role as the mean optical depth of the layer grows. The most rigorous constraints actually occur for optical thicknesses ∿1, because the contribution of deeper layers is ordinarily small in the departing radiation.

The dependences of the absorption coefficient and the Planck function on the temperature fluctuations can be represented approximately in the form

$$k = k_0 \exp(\alpha t), \tag{14}$$

$$B = B_0 \exp(\beta t), \tag{15}$$

where α and β are, respectively, the ratio between the transition excitation energy and the quantum energy expressed in degrees, and the mean temperature, and t is the dimensionless (referred to the mean temperature) temperature fluctuations. Taking the normal distribution law for t, we obtain from (12) and (13) for a homogeneous layer of thickness L for $\langle k \rangle L \sim 1$:

$$l \exp(\alpha^2 \langle t^2 \rangle) \ll L, \tag{16}$$

$$l \exp(\alpha(\alpha + \beta) \langle t^2 \rangle) \ll L. \tag{17}$$

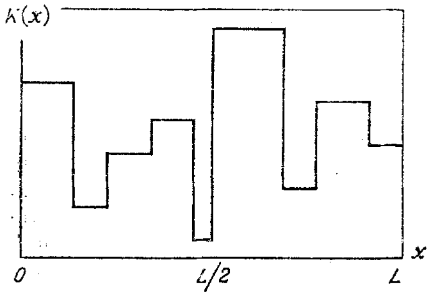

Fig. 1. One possible realization of the spatial realization of the absorption coefficient.

Taking into account that $l \sim 0.1L$, it can be concluded from (16), (17) that the OFTA is valid for the computation of the radiation of volumes with optical depth not exceeding several units, for $\alpha(\alpha + \beta)\langle t^2\rangle \leqslant 1$. It is hence seen that the maximal value of α for which the OFTA is still applicable is reached for $\beta = 0$ and equals $\alpha_{max} = \langle t^2\rangle^{-1/2}$. For 20% temperature pulsations $\alpha_{max} = 5$. The values of β can be arbitrary. However, as β grows, the values of α allowable from the viewpoint of the applicability of the OTFA are diminished in conformity with (17).

On the wings of the molecular bands where absorption is due to transitions from an excited vibrational level, it follows from the resonance condition that $\alpha \simeq \beta$. At the maximum of the dependence of the Planck function on the wavelength, $\beta = 5$. Therefore, the values of α and β of interest can reach 10-15, which can results in spoilage of the condition of OTFA applicability for the typical quantity $\sqrt{\langle t^2\rangle} = 0.2$ and sufficiently large optical depths of the layer. Consequently, a computation of the mean radiation of turbulent flows for large fluctuations in the absorption coefficient and the Planck function is an urgent problem.

Since its exact solution is impossible at this time, models of radiating media that permit an analytic solution to be obtained for arbitrary dependences of the Planck function and the absorption coefficient on the temperature play an important part in clarification of the qualitative pattern of the phenomenon as well as in setting up the limits of applicability of approximate methods. Let us examine the model of a medium in which the absorption coefficient depends only on the temperature and is described by a generalized telegraph process [10]:

$$k(x) = \varkappa_{n(0,x)}, \tag{18}$$

where both the subscript n(0, x) representing the number of points of the Poisson flux incident in the interval [0, x], and the quantities $\varkappa_i$ for fixed subscripts are random. The quantities $\varkappa_i$ are statistically independent and characterized by distribution functions independent of the subscript. The absorption coefficient profile (18) is a sequence of sections of constant random values. The lengths of the sections are also random. It is natural to select the mean length of one section equal to l. One of the possible realizations of the absorption coefficient profile is represented in Fig. 1.

The theory of a generalized telegraph process results in [10]

$$\begin{aligned}\left\langle k(t(x))\,B(t(x))\exp\left(-\int_0^x k(y)\,dy\right)\right\rangle &= \langle \varkappa B(\varkappa)\exp[-(\varkappa + l^{-1})x]\rangle + \\ &+ l^{-1}\int_0^x \langle \varkappa B(\varkappa)\exp[-(\varkappa + l^{-1})(x-y)]\rangle\,\Phi(y)\,dy,\end{aligned} \tag{19}$$

where the mean transmission Φ is determined by (19) for kB = 1:

$$\Phi(x) = \langle \exp[-(\varkappa + l^{-1})x]\rangle + l^{-1}\int_0^x \langle \exp[-(\varkappa + l^{-1})(x-y)]\rangle\,\Phi(y)\,dy. \tag{20}$$

Integrating (19) with respect to x between 0 and L, we obtain an expression for the mean brightness of the layer

$$\begin{aligned}\langle I\rangle &= \langle \varkappa B(\varkappa)(\varkappa + l^{-1})^{-1}[1 - \exp[-(\varkappa + l^{-1})L]]\rangle + \\ &+ l^{-1}\int_0^L \Phi(x)\langle \varkappa B(\varkappa)(\varkappa + l^{-1})^{-1}[1 - \exp[-(\varkappa + l^{-1})(L-x)]]\rangle\,dx.\end{aligned} \tag{21}$$

Analysis of the Laplace transform of the solution of the integral equation (20)

$$f(\lambda)=\int_0^\infty \exp(-\lambda x)\Phi(x)\,dx=\left[\lambda+\left\langle\frac{\varkappa}{\varkappa+l^{-1}+\lambda}\right\rangle\Big/\left\langle\frac{1}{\varkappa+l^{-1}+\lambda}\right\rangle\right]^{-1} \qquad (22)$$

shows that in the case of fundamental interest of statistical transmissivity of an individual pulsation, whose condition is the inequality

$$\langle \varkappa(\varkappa+l^{-1})^{-1}\rangle \ll 1, \qquad (23)$$

the asymptotic $\Phi(x)$ has the following form for $x \gg l$

$$\Phi(x)=\exp[-\langle \varkappa x(1+\varkappa l)^{-1}\rangle]. \qquad (24)$$

Substituting (24) into (21) and taking account of the inequalities (23) and (2), we obtain an expression for the mean brightness

$$\langle I\rangle=\left\langle\frac{\varkappa Bl}{1+\varkappa l}\right\rangle\left\langle\frac{\varkappa l}{1+\varkappa l}\right\rangle^{-1}\left[1-\exp\left[-N\left\langle\frac{\varkappa l}{1+\varkappa l}\right\rangle\right]\right], \qquad (25)$$

where $N = L/l$ is the mean number of pulsations per light path.

The physical meaning of the quantities in (25) can be clarified by using the exponential distribution law for the spacing between adjacent points of the Poisson flux. The transmission of a single homogeneous section averaged over the length equals $(1 + \varkappa l)^{-1}$. Therefore, the quantity $<\varkappa l(1 + \varkappa l)^{-1}>$ has the meaning of average absorptivity of a homogeneous section, or one pulsation. The spatial correlation of the fluctuating quantities in turbulent media results in the fact that a rise in emissivity of a certain volume element is accompanied by an increase in absorption in its neighborhood. This circumstance is reflected in the quantity $\Delta x<\varkappa B(1 + \varkappa l)^{-1}>$, which is the mean emission being generated per length element Δx and passed through a homogeneous section with temperature equal to the temperature of the emitting volume. The quantity $<\varkappa lB(1 + \varkappa l)^{-1}>$ that reflects the contribution of a section of length l to the mean emission of the layer can be interpreted as the mean brightness of a pulsation.

The OFTA (3) follows from (25) if the quantity $\varkappa l$ can be neglected in the denominators of the expressions for the absorptivity and brightness of one pulsation

$$\langle I\rangle=\langle \varkappa B\rangle\langle\varkappa\rangle^{-1}[1-\exp(-\langle\varkappa\rangle L)]. \qquad (26)$$

The conditions for going from (25) to (26) agree with (12) and (13).

Let us turn to the case of large fluctuations in the absorption coefficient and Planck function, and let us consider how the mean radiation changes as the concentration of the emitting substance grows. The condition of statistical transparency of the layer

$$N\langle \varkappa l(1+\varkappa l)^{-1}\rangle \ll 1 \qquad (27)$$

is satisfied within the limit of vanishingly small concentrations and the equality

$$\langle \varkappa Bl(1+\varkappa l)^{-1}\rangle \simeq \langle \varkappa Bl\rangle \qquad (28)$$

is valid, resulting in the expression

$$\langle I\rangle=N\langle\varkappa Bl\rangle, \qquad (29)$$

which is linearly dependent on the concentration and in agreement with the corresponding OFTA limit case because of satisfaction of conditions (12) and (13).

As the concentration grows in a medium with strongly fluctuating optical characteristics, the equality (28) is spoiled earlier than the condition (27) since the sharp temperature dependence of the Planck function increases the contribution, as compared with (27), of the high temperature values to the averaged quantities. In this case the dependence of the emission on the concentration becomes weaker as compared with the linear dependence

$$\langle I\rangle=N\langle\varkappa Bl(1+\varkappa l)^{-1}\rangle, \qquad (30)$$

which is a result of the correlation between the emission at a point and the absorption in its neighborhood.

As the concentration grows further, the fraction of time during which the pulsation remains in the state of opacity increases. Two cases can here be realized, depending on which occurs earlier, saturation of the emission of a single pulsation or spoilage of condition (27). For sufficiently sharp dependences of the Planck function and absorption coefficient on the temperature, when the fundamental contribution to the emission by one pulsation is given by temperature fluctuations for which the pulsation is opaque and emits as a black body, the first case holds. As estimates show, on the basis of (14), (15) and the normal distribution law of the temperature pulsations, the layer statistical transmissivity, whose condition (27) weakly depends logarithmically on N for $\alpha\sqrt{\langle t^2\rangle} > 1$, holds for N = 10 for

$$\ln k_0 l \leqslant -1.8\alpha\sqrt{\langle t^2\rangle}, \tag{31}$$

while the saturation of the emission of one pulsation occurs for

$$\ln k_0 l \geqslant -\alpha\beta\langle t^2\rangle + 1.8\alpha\sqrt{\langle t^2\rangle}. \tag{32}$$

In this case the mean emission of one pulsation equals the mean of the Planck function while the total layer emission ceases to depend on the concentration:

$$\langle I\rangle = N\langle B\rangle. \tag{33}$$

A further increase in the concentration results in spoilage of inequality (27). Taking account of the smallness of the exponential term in (25), we obtain that in the case of saturation of one pulsation

$$\langle I\rangle = \langle B\rangle[\langle \varkappa l(1+\varkappa l)^{-1}\rangle]^{-1} \simeq \langle B\rangle n^*, \tag{34}$$

where n^* is the number of pulsations whose emission reaches the layer boundary without noticeable absorption, defined by the condition

$$n^*\langle \varkappa l(1+\varkappa l)^{-1}\rangle \sim 1. \tag{35}$$

Therefore, if saturation of the emission of a single pulsation precedes spoilage of the condition of statistical transparency of the layer (27), then as the concentration grows the emission grows according to (29), (30), emerges on a plateau of limit values (33), and then drops in conformity with (34) at the instant of spoilage of the statistical transparency of the layer because of diminution in the number of pulsations inducing a contribution to the emerging emission.

If spoilage of the statistical transmissivity condition (27) occurs before saturation of the emission of individual pulsations, then the mean emission is defined by (30) while condition (27) is valid, and then can be represented by an expression analogous to (30) but with N replaced by n^*. In this case the plateau of the limit values degenerates into a maximum whose magnitude is less than (33), while the location is governed by the competition of two factors, the growth of the emission of one pulsation and the diminution of layer transmission.

Common to all the cases considered, starting with (29), is the fact that the mean emission of the layer is represented in the form of the product of the emission of one isolated pulsation by their number defined by the depth of layer transmissivity. Consequently, the regime of mean emission formation examined above can be called the isolated pulsation regime.

As the concentration increases further without limit, the quantum path becomes much smaller than the dimension of the pulsation and the layer emission defined by the surface temperature in this case becomes equal to

$$\langle I\rangle = \langle B\rangle. \tag{36}$$

This result is obtained from the expression (21), which is more general than (25), when taking account of the circumstance that the transmissivity Φ differs from zero just for values of the argument much less than l.

A number of the fundamental features of the qualitative dependence of the mean emission on the concentration described above, particularly the isolated pulsation regime, can be obtained without reliance on model representations. Taking the approximate average of the expression (4) for the brightness with (2) taken into account for not too strongly absorbing media when the contribution of a layer of thickness $\sim l$ can be neglected in the total emission of the volume, we obtain

$$\langle I\rangle \simeq \int_l^L \left\langle kB\exp\left(-\int_{x-l}^{x} k dy\right)\right\rangle \left\langle \exp\left(-\int_0^{x-l} k dy\right)\right\rangle dx. \tag{37}$$

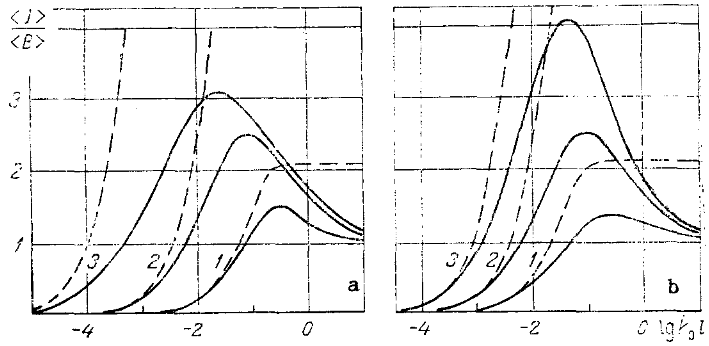

Fig. 2. Mean thermal emission of a plane layer for different values of the parameters α and β. Solid lines are a computation using (25) and the dashes by using (26): a) β = 7.5; 1) α = 2.5; 2) 7.5; 3) 12.5; b) α = 7.5; 1) β = 2.5; 2) 7.5; 3) 12.5.

Taking account of the weak dependence of the first factor in the integrand on the coordinates, we write (37) in a form analogous to (25):

$$\langle I \rangle \simeq \left\langle kB \exp\left(-\int_0^l k dy\right)\right\rangle \int_l^L \left\langle \exp\left(-\int_0^{x-l} k dy\right)\right\rangle dx. \tag{38}$$

For sufficiently low concentrations when the layer is statistically transmissive and the integral in (37) equals L, we obtain an expression analogous to (32):

$$\langle I \rangle \simeq N \left\langle kBl \exp\left(-\int_0^l k dy\right)\right\rangle. \tag{39}$$

It follows from (38) that upon spoilage of the statistical transmissivity, a formula of the type (39) with N replaced by n^* is valid, where ln^* is the depth to which the layer is, on the average, transmissive, and equal to the value of the integral in (38). Therefore, the existence of an isolated pulsations regime is associated with the smallness of the correlation length as compared with the layer thickness and not with the selection of a specific model of the medium.

Numerical computations of the emission were performed by using the dependences (14), (15) and the normal distribution function of the temperature fluctuations. The relationship between the size of the emitting layer L and the external scale of turbulence l, as well as the amplitude of the temperature fluctuations were selected as typical for a turbulent jet: $N = L/l = 10$, $\sqrt{\langle t^2 \rangle} = 0.2$. In particular, there follows from the computation results that the plateau of limit values (38) in the dependence of the mean emission on the concentration is realized for $\alpha \geqslant 10$, $\beta \geqslant 20$. As the parameters α and β diminish, the plateau is transformed into a maximum whose magnitude is less than the value determined by (38). This maximum vanishes completely as α or β tend to zero.

Computed dependences of the mean emission on the optical depth of one pulsation $k_0 l$, quantities proportional to the concentration, are represented in Fig. 2. As is seen from Fig. 2, the dependences of the emission on $k_0 l$ computed by means of (25) and (26) are distinctive not only quantitatively but also qualitatively. The emission in the OFTA increases more rapidly than (25) and tends monotonically to the limit $\langle \varkappa B \rangle / \langle \varkappa \rangle$ in contrast to the dependence (25) which passes through a maximum and tends to the natural limit $\langle B \rangle$ as the optical depth $k_0 l$ increases further. The slower growth of (25) as compared with (26) is due to the correlation between the emission of a volume element and the absorption in its neighborhood, which is not taken into account in the OFTA.

Therefore, the results obtained in this paper can be applied to compute the emission of molecular gas jets in absorption bands and isolated spectrum lines, in particular, where the main emission occurs at frequencies with quantum path length on the order of the jet transverse dimension, and therefore, the constraints on applicability of OFTA (12), (13) are most rigorous.

LITERATURE CITED

1. V. M. Ievlev, Turbulent Motion of High-Temperature Continuous Media [in Russian], Nauka, Moscow (1975).

2. V. A. Baryshev, "On the influence of random atmosphere inhomogeneities on the intensity of earth's departing thermal radiation," Problems of Physics of the Atmosphere [in Russian], No. 3, Leningrad State Univ. (1963), pp. 105-110.
3. R. M. Kowalik and C. H. Kruger, "The effect of fluctuations and nonuniformities on line-reversal temperature measurements," JQSRT, 18, No. 5, 627-636 (1977).
4. J. W. Dayly, "Effect of turbulence on line-reversal temperature measurements," JQSRT, 17, No. 3, 339-341 (1977).
5. V. P. Kabashnikov and G. I. Kmit, "Influence of turbulent pulsations on thermal radiation," Zh. Prikl. Spektrosk., 31, No. 2, 226-231 (1979).
6. V. P. Kabashnikov and G. I. Kmit, "Taking account of the influence of turbulent pulsations on the thermal radiation of a medium in a square approximation," Inzh.-Fiz. Zh., 37, No. 3, 405-411 (1979).
7. J. S. Draper, L. S. Bernstein, and W. K. Cheng, "Fluctuating emission from molecular vibration—rotational bands," JQSRT, 23, No. 3, 323-326 (1980).
8. P. Gouttenbroze, "On the effect of temperature fluctuations on line intensities," JQSRT, 30, No. 3, 193-211 (1983).
9. Yu. A. Popov, "On the influence of temperature pulsations on flame radiation and the spectrum line-inversion method," Teplofiz. Vys. Temp., 18, No. 2, 383-386 (1980).
10. V. I. Klyatskin, Stochastic Equations and Waves in Randomly Inhomogeneous Media [in Russian], Nauka, Moscow (1980).

NONUNIFORMITY OF THE VELOCITY FIELD OF A FLUX PASSING THROUGH A PACKED BED

M. A. Gol'dshtik, A. M. Vaisman,
A. V. Lebedev, and M. Kh. Pravdina

UDC 532.546.2

It is experimentally and theoretically shown that the sharp nonuniformity of the velocity at the outlet from a packed bed develops outside the bed as the flow passes through a curvilinear boundary.

The strongly pronounced nonlocalized velocity nonuniformity in a flow emerging from a packed bed has been investigated experimentally [1-5] and theoretically [4, 6-8] for a period of more than over 20 years. However, complete clarity with regard to the character and nature of this phenomenon has not yet been achieved. Experiments indicate that the nonuniformity scale is more likely connected with the channel dimensions than with the bead diameter. Theoretical investigations are based on the assumption that the velocity nonuniformity develops within the packed bed due to changes in its porosity, caused by repacking or deformations. For all the diversity of the deformation models used, the interaction between the bed and the channel walls plays the central role. An alternative approach is based on the possibility that the deflection of the supporting grid may be the cause of velocity nonuniformity [5]. This possibility has not been investigated to a sufficient extent. Therefore, we have performed experiments in order to compare the effect of the walls with that of the supporting grid.

The device for blowing air through a bed of beads (Fig. 1a) makes it possible to vary the deflection of the supporting grid and also introduce an additional wall, not connected to the grid, in the middle of the channel (Fig. 1b). The device consists of a vertical, rectangular channel with a 120 × 60 mm cross section, which has three parts: the supply section 5, the operating section 6, and the outlet channel 7. The endfaces of the channel sections have flanges with rubber gaskets providing an airtight seal. Air is supplied to the channel from the main at pressures of up to 8 atm through branch pipe 1 and is then transmitted through swirler 2 and equalizer 3. In spite of its small dimensions, this inlet arrangement ensures relatively good equalization of the air flow ahead of the bead bed

Institute of Thermophysics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 1, pp. 42-51, July, 1985. Original article submitted July 4, 1984.

 0022-0841/85/4901-0784$09.50